Q - State and prove theorem of greatest lower bound.

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Statement -

Any non-empty set of real numbers which is bounded below has greatest lower bound.

Proof - Let X be a non empty set of real numbers which is bounded below and let Y be the set of lower bounds for X.

Let $c \in X$ Then $y \leq c \; \forall \; y \in Y$

Thus Y is bounded above and hence by the theorem of least upper bound Y has a least upper bound a. We will show that a is the greatest lower bound of X.

Let $x \in X$. Then $y \leq x$ for all $y \in Y$ and thus x is an upper bound for Y. Since a is the least upper bound of Y, we have $a \leq x$.

Therefore, a is a lower bound for X.

Let b be any lower bound for X.

Then $b \in Y$ and hence $b \leq a$.

Hence by definition, a is the greatest lower bound of X.

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